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APPROXIMATION OF THE TIME FORM OF A LASER PULSE USING FINITE FUNCTIONS

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The influence of the time form of a surface heat source on the distribution of the temperature field in a homogeneous half-space is investigated.

Introduction. The intensity of laser radiation is either constant (continuous operation) or a function of time (pulsed operation) [1]. Most gas lasers, in particular, CO_2 and He–Ne ones, can operate in both continuous and pulsed lasing regimes. Solid-state lasers (ruby, neodymium glass, and Nd:YAG ones) are used in a pulsed regime, although the latter can operate continuously, too. The function describing the time variation in the specific laser-radiation power is called the time form (structure) of a laser beam. The time structure of a pulse of modern lasers can vary with operating conditions (e.g., ruby and neodymium lasers).

We have a millisecond (normal) lasing regime when the laser is pumped using a flash lamp. The pulse duration typical of this operating regime of the laser takes on values in the interval 0.1–1 msec. Usually, such a pulse consists, in turn, of a series of randomly occurring flashes with a duration of about a few microseconds. The amplitude and time intervals between them are different. Ruby and neodymium glass lasers frequently operate in such a regime. In a millisecond regime of laser operation, we can also have the generation of a quasistationary pulse, when microflashes are absent. A typical oscillogram of variation in the specific power of a neodymium glass laser operating in a millisecond regime is shown in Fig. 1 [2].

Heat-Conduction Problem. The depth of penetration of laser light into the irradiated material is much smaller than the thickness of a layer heated by heat conduction. It is well known that if the thickness of the heated layer is much smaller than the radius of the laser beam, the process of heating of a body can be modeled using a surface heat source [1]. In selecting laser-radiation parameters necessary for forming the temperature field of a certain (for this depth) level in a material with prescribed thermophysical properties, one uses, as a rule, the solution of a one-dimensional (in space coordinate) linear heat-conduction equation for a semiinfinite body $z \geq 0$ [3]:

$$T(z, t) = \frac{Aq_0}{K} \sqrt{\frac{k}{\pi}} \int_0^t \frac{I(s)}{\sqrt{t-s}} \exp\left[-\frac{z^2}{4k(t-s)}\right] ds , \quad z \geq 0 , \quad t \geq 0 . \tag{1}$$

In numerical calculations of laser heating, one usually considers either the rectangular

$$I(t) = H(t_s - t) , \quad t > 0 , \tag{2}$$

or triangular

$$I(t) = \begin{cases} 2t/t_r , & 0 < t \leq t_r ; \\ 2(t_s - t)/(t_s - t_r) ; & t_r \leq t \leq t_s , \end{cases} \tag{3}$$

structures of a laser pulse, selecting their parameters for comparative analysis so that the duration and energy are equal in both cases.

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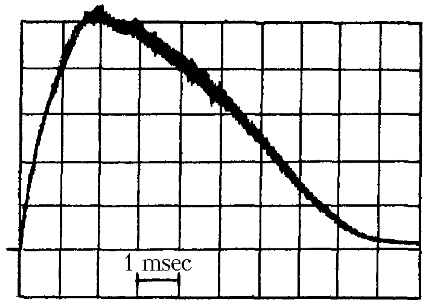


Fig. 1. Oscillogram of the radiation-intensity distribution of a laser operating in the quasistationary regime of radiation (from the data of [2]).

Passing to dimensionless quantities and parameters

$$\zeta = \frac{z}{a} , \quad \tau = \frac{kt}{a^2} , \quad \tau_r = \frac{kt_r}{a^2} , \quad \tau_s = \frac{kt_s}{a^2} , \quad T^* = \frac{T}{\Lambda} , \quad \Lambda = \frac{Aq_0 a}{K} , \tag{4}$$

after the substitution of the functions of I (2) and (3) into the solution (1) and the integration, we obtain

$$T(\zeta, \tau) = A\Lambda T^* (\zeta, \tau) , \quad \zeta \geq 0 , \quad \tau \geq 0 , \tag{5}$$

where we have

$$T^* (\zeta, \tau) = T^{(0)*} (\zeta, \tau) - T^{(0)*} (\zeta, \tau - \tau_s) , \tag{6}$$

$$T^{(0)*} (\zeta, \tau) = 2 \sqrt{\tau} \ \text{ierfc} \ (0.5\zeta / \sqrt{\tau}) \ H(\tau) , \tag{7}$$

for the rectangular form of the pulse (2) and

$$T^* (\zeta, \tau) = \frac{2}{\tau_r} \left[T^{(1)*} (\zeta, \tau) - T^{(1)*} (\zeta, \tau - \tau_r) \right] - \frac{2}{\tau_s - \tau_r} \left[T^{(1)*} (\zeta, \tau - \tau_r) - T^{(1)*} (\zeta, \tau - \tau_s) \right] , \tag{8}$$

$$T^{(1)*} (\zeta, \tau) = 2\tau \sqrt{\tau} \left\{ \left[1 + \frac{2}{3} \left(\frac{\zeta}{2\sqrt{\tau}} \right)^2 \right] \text{ierfc} \left(\frac{\zeta}{2\sqrt{\tau}} \right) - \frac{1}{3\sqrt{\pi}} \exp \left(- \frac{\zeta^2}{4\tau} \right) \right\} H(\tau) \tag{9}$$

for the triangular form (3).

It is noteworthy that $T^{(0)*}$ and $T^{(1)*}$ are the known solutions obtained from formula (1) for a constant $I(\tau)$ = 1 and a linear $I(\tau) = \tau$, $\tau > 0$, of the specific power of the surface heat source [3].

An approximation of the oscillogram presented in Fig. 1, which is more accurate than those rectangular and triangular, can be obtained using the function [4]

$$I(\tau) = I^* \exp \left[-\beta \left(\tau^\gamma - \tau_r^\gamma \right) \right] \left(\tau / \tau_r \right)^\alpha , \quad \tau > 0 , \tag{10}$$

where the parameters α, β, and γ are related to the rise time

$$\tau_r = [\alpha / (\beta\gamma)]^{1/\gamma} . \tag{11}$$

The dimensionless factor I^* in formula (10) is selected from the condition of equality of the total energy for the distributions (2), (3), and (10):

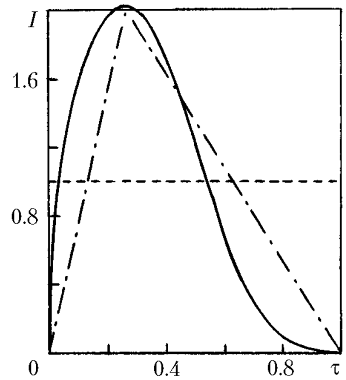


Fig. 2. Different forms of the time structure of a laser pulse.

$$\int_0^\infty I(\tau)\, d\tau = \tau_s . \tag{12}$$

In particular, when $\alpha = 0.4$, $\beta = 7$, and $\gamma = 3$ we find $\tau_r = 0.267$ from equality (11) and relation (12) yields $I^* = 2.019\tau_s$. The plot of the function $I(\tau)$ (10) constructed for $\tau_s = 1$ is presented in Fig. 2 by a solid curve. The rectangular (2) (dashed curve) and triangular (3) (dot-dash curve) distributions of the heat-flux intensity are also shown in this figure.

We failed to accurately integrate the solution (1) for the function $I(\tau)$ (10) and (11). Therefore, we perform integration, approximating the function $I(\tau)$ (10) using finite piecewise-continuous or piecewise-linear functions. Such an approach turned out to be efficient earlier in investigating a two-dimensional quasistationary heat-conduction problem for a half-space heated by a rapidly moving distributed linear heat flux (Ling thermal friction problem) [5, 6].

In the case of the piecewise-continuous approximation, we have

$$I(\tau) \approx \sum_{k=1}^{n} I(\bar{\tau}_k)\, \varphi_k(\tau), \quad \tau > 0 , \tag{13}$$

$$\varphi_k(\tau) = \begin{cases} 1 , & \tau \in [\tau_{k-1}, \tau_k] , \\ 0 , & \tau \notin [\tau_{k-1}, \tau_k] , \quad k = 1, 2, ..., n , \end{cases} \tag{14}$$

where $\tau_k = k\delta\tau$ and $\bar{\tau}_k = 0.5\,(\tau_{k-1} + \tau_k)$, $k = 0, 1, ..., n$; $\delta\tau = \tau/n$ are the nodes of the grid of uniform partition of the time interval $[0, \tau]$.

Substituting the approximation of the $I(\tau)$ function (13) and (14) into the solution (10) and integrating, we obtain the solution in the form (5), where

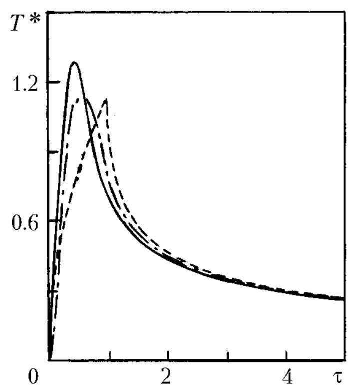
$$T^*(\zeta, \tau) = \sum_{k=1}^{n} I(\bar{\tau}_k)\, T_k^{(0)*}(\zeta, \tau) , \tag{15}$$

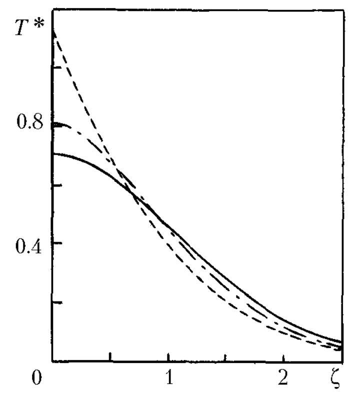
$$T^{(0)*}(\zeta, \tau) = T^{(0)*}(\zeta, \tau - \tau_{k-1}) - T^{(0)*}(\zeta, \tau - \tau_k) ; \tag{16}$$

here $T^{(0)*}(\zeta, \tau)$ is the dimensionless temperature (7).

Approximating the time structure $I(\tau)$ (10) by the piecewise-linear functions

$$I(\tau) \approx \sum_{k=0}^{n} I(\tau_k)\, \varphi_k(\tau), \quad \tau > 0 , \tag{17}$$





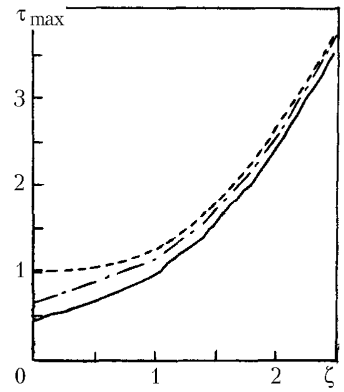


Fig. 3. Evolution of the dimensionless temperature T^* on the half-space surface: $\tau_s = 1$ and $\tau_r = 0.267$.

Fig. 4. Distribution of the dimensionless temperature T^* in depth from the half-space surface. $\tau = \tau_s = 1$ and $\tau_r = 0.267$.

Fig. 5. Change in the dimensionless time τ_{max} of attainment of the maximum value by the temperature with distance from the half-space surface. $\tau_s = 1$ and $\tau_r = 0.267$.

$$\varphi_0(\tau) = \begin{cases} (\tau_1 - \tau)/\delta\tau \,, & \tau \in [\tau_0, \tau_1] \,; \\ 0 \,, & \tau \notin [\tau_0; \tau_1] \,, \end{cases} \qquad \varphi_n(\tau) = \begin{cases} (\tau - \tau_{n-1})/\delta\tau \,, & \tau \in [\tau_{n-1}, \tau_n] \,; \\ 0 \,, & \tau \notin [\tau_{n-1}, \tau_n] \,; \end{cases}$$

$$\varphi_k(\tau) = \begin{cases} (\tau - \tau_{k-1})/\delta\tau \,, & \tau \in [\tau_{k-1}, \tau_k] \,; \\ (\tau_{k+1} - \tau)/\delta\tau \,, & \tau \in [\tau_k, \tau_{k+1}] \,; \\ 0 \,, & \tau \notin [\tau_k, \tau_{k+1}] \,; \end{cases}$$

from the solution (1) we obtain the temperature in the form (5)

$$T^*(\zeta, \tau) = \frac{1}{\delta\tau} \sum_{k=0}^{n} I(\tau_k)\, T_k^{(1)*}(\zeta, \tau) \,, \tag{18}$$

$$T_0^{(1)*}(\zeta, \tau) = \tau_1 T^{(0)*}(\zeta, \tau) - T^{(1)*}(\zeta, \tau) + T^{(1)*}(\zeta, \tau - \tau_1) \,,$$

$$T_n^{(1)*}(\zeta, \tau) = T^{(1)*}(\zeta, \tau - \tau_{n-1}) - T^{(1)*}(\zeta, \tau - \tau_n) - (\tau_n - \tau_{n-1})\, T^{(0)*}(\zeta, \tau - \tau_n) \,,$$

$$T_k^{(1)*}(\zeta, \tau) = T^{(1)*}(\zeta, \tau - \tau_{k-1}) - 2T^{(1)*}(\zeta, \tau - \tau_k) + T^{(1)*}(\zeta, \tau - \tau_{k+1}) \,, \quad k = 1, 2, ..., n-1 \,,$$

where $T^{(1)*}(\zeta, \tau)$ is the dimensionless temperature (9).

The absolute error of approximations (13) and (14) is $O(\delta\tau)$ and $O(\delta\tau^2)$ respectively [7].

Numerical Analysis. The evolution of the dimensionless temperature $T^* = T/\Lambda$ on the irradiated surface $\zeta = 0$ for the rectangular (2), triangular (3), and accurate (10), (11) forms of the time distribution of the specific power of the heat source is shown in Fig. 3. Here and in Figs. 4 and 5, the solid curves correspond to the nearly triangular distribution of radiation intensity (10) and (11), the dot-dash curves correspond to the triangular distribution (3), and the dashed curves correspond to the rectangular one (2).

For the rectangular pulse, the maximum value $T^*_{max} = 1.1284$ is attained at the instant of switching the source off $\tau_s = 1$, whereas for the triangular pulse of the same duration with a rise time $\tau_r = 0.2671$, the maximum value is

T^*_{max} = 1.1429 at τ = 0.58. We have an even larger shift of the time of attainment of the maximum temperature from the instant of switching the source off for the distribution of the heat flux intensity in the form (10) and (11) (with the same rise time), when T^*_{max} = 1.2824 at τ = 0.46.

The variation in the temperature with depth from the heating surface is shown in Fig. 4. The interior of the body is cooled most rapidly for the rectangular form of a pulse, and the cooling is the slowest for the intensity distribution (10). The temperature becomes negligibly low at a distance of about three radii of the heat source for all three time structures.

The effect of delay is also noteworthy: the time τ_{max} of attainment of the maximum temperature grows with distance from the working surface [8]. For the rectangular pulse we have $\tau_{max} = \tau_s + \Delta\tau$. The delay time on the body's surface ζ = 0 is $\Delta\tau$ = 0 and it increases with distance from the surface. For the triangular (3) and nearly triangular (10) time forms of a laser pulse, we have a delay from the rise time, i.e., $\tau_{max} = \tau_r + \Delta\tau$. For a fixed depth ζ, the time of attainment of the temperature maximum is the longest for the rectangular pulse and the shortest for the time distribution of the specific radiation power (10) (Fig. 5). For example, at the depth ζ = 1, we have τ_{max} = 1.26 for the rectangular pulse, τ_{max} = 1.15 for the triangular pulse, and τ_{max} = 0.97 for the pulse described by formulas (10) and (11).

Influence of the Time Form of a Laser Pulse on the Thickness of a Thermally Hardened Layer. Pulsed laser treatment gives rise to a hardened layer of small (about 0.1 mm) thickness near the irradiated surface [9]. Apart from the power and duration of a laser pulse, a substantial influence on the thickness of the hardened layer is exerted by its evolution with time. Since, mathematically, the form is most simply varied using the ratio τ_r/τ_s, we comparatively investigate the corresponding parameters for the rectangular (2) and triangular (3) forms of a laser pulse.

According to the solution (6) and (7), at the instant of switching off $\tau = \tau_s$, the dimensionless temperature at the distance ζ from the body's surface is equal to

$$T^*_0(\zeta, \tau_s) = 2\sqrt{\tau_s}\,\mathrm{ierfc}\,(0.5\zeta\sqrt{\tau_s})\,, \quad \zeta > 0 \tag{19}$$

for the rectangular pulse (2) and, based on the solution (8) and (9), to

$$T^*_1(\zeta, \tau_s) = \frac{2}{\tau_r}\left[T^{(1)*}(\zeta, \tau_s) - \frac{\tau_s}{\tau_s - \tau_r}T^{(1)*}(\zeta, \tau_s - \tau_r)\right], \quad \zeta > 0 \tag{20}$$

for the triangular pulse (3).

On the surface irradiated, from formulas (19) and (20) for ζ = 0 we obtain

$$T^*_0(\zeta, \tau_s) = 2\sqrt{\tau_s}\,, \quad T^{(1)}_1(0, \tau_s) = 8\sqrt{\tau_s}/[3\sqrt{\pi}\,(1 + \sqrt{1 - \tau_r/\tau_s})]\,. \tag{21}$$

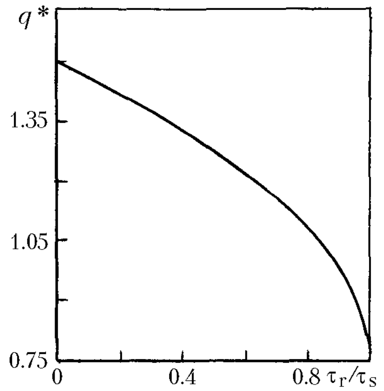
Let us assume that the parameters of a laser unit are such that at the instant of switching the unit off, the temperature on the surface of the irradiated body is the same and equal to T_0 for the rectangular and triangular pulse forms, i.e., we have

$$A\Lambda_0 T^*_0(0, \tau_s) = A\Lambda_1 T^*_1(0, \tau_s) = T_0\,. \tag{22}$$

For the same absorption coefficients A and beam radius a, from equality (22) we find the ratio of the specific powers of the rectangular and triangular heat sources

$$q^* = \frac{q_1}{q_0} = \frac{T^*_0(0, \tau_s)}{T^*_1(0, \tau_s)}\,, \tag{23}$$

where the dimensionless temperatures T^*_0 and T^*_1 are determined from formulas (21). The dependence of q^* (23) on the ratios τ_r/τ_s is shown in Fig. 6. It is seen that the intensity of the triangular pulse with a maximum close to the instant of switching the laser off and zero at the instant of switching it on (τ_r/τ_s = 0), which is necessary for attainment of



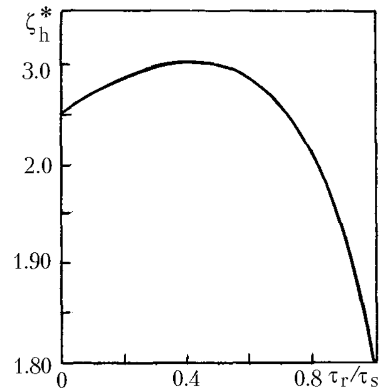


Fig. 6. Ratio $q^* = q_1/q_0$ of the specific powers of the heat sources of triangular and rectangular time forms vs. dimensionless rise time τ_r/τ_s.

Fig. 7. Ratio $\zeta_h^* = \zeta_h^{(1)}/\zeta_h^{(0)}$ of the dimensionless hardened-layer thicknesses formed in heating of the body by the sources of triangular and rectangular time forms vs. dimensionless rise time τ_r/τ_s at $T_h/T_0 = 0.666$.

a prescribed temperature on the body's surface, is half again the specific power of the beam of the rectangular form. As the ratio τ_r/τ_s grows, the quantity q^* decreases, attaining a value of 0.75 for the pulse with zero at the leading edge and a maximum at the trailing edge ($\tau_r/\tau_s = 1$). This is confirmed by the conclusion of [10] that, in solving technical problems associated with the melting and evaporation of a material, the action by triangular pulses with a low-angle leading edge and a steep trailing edge is the most efficient from the viewpoint of the minimum energy consumption.

Knowing the temperature of laser hardening T_h with the condition (20) of equality of the surface temperatures of the rectangular and triangular pulses, from relations (5), (19), and (20) we obtain the equations to determine the dimensionless thickness ζ_h of the hardened layer

$$T_k^*(\zeta_{(h)}^{(0)}, \tau_k) = (T_h/T_0)\, T_k^*(0, \tau_1)\,,$$
$$T_k^*(\zeta_h^{(1)}, \tau_k) = (T_h/T_0)\, T_k^*(0, \tau_1)\,. \qquad (24)$$

The dependence of the ratio $\zeta_h^* = \zeta_h^{(1)}/\zeta_h^{(0)}$ on the ratio τ_r/τ_s is shown in Fig. 7. It is seen that, on condition that a constant temperature is maintained on the surface irradiated, for the largest thickness of the hardened layer to be produced one must carry out the treatment by triangular pulses with an intensity maximum falling nearly at the middle of the pulse duration (in the case in question, the rise time is $\tau_r = 0.43\tau_s$). Thus, the time form of a pulse that can be approximated by an isosceles triangle is optimum from the viewpoint of production of the largest hardened-layer thickness without changing the state of the surface.

Conclusions. Using the approximating properties of finite functions we have considered the approximate solution of a one-dimensional nonstationary heat-conduction problem for a semiinfinite body with an arbitrary (integrable) distribution of the heat-flux intensity on the boundary surface. The numerical calculations have been carried out for the same form (occurring in pulsed laser treatment of materials) of the specific power of a surface heat source. A comparative analysis of the temperature distribution for pulses of rectangular, triangular, and "nearly triangular" time structures with the same power has been presented. Also, we have investigated the influence of different time forms of a laser pulse on the hardened-layer thickness.

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NOTATION

A, absorption coefficient; a, radius of the laser beam; $\text{erfc}(\cdot) = 1 - \text{erf}(\cdot)$; $\text{erf}(\cdot)$, error function; $H(\cdot)$, Heaviside unit function; K and k, thermal conductivity and thermal diffusivity respectively; $q = Aq_0I$, radiation intensity; q_0, characteristic value of q; I, time structure of a pulse; T, temperature; $T^* = T/\Lambda$, dimensionless temperature; T_h, temperature of thermal hardening of steel; t, time; t_s, duration of heating; t_r, rise time; Δt, delay time; z, space coordinate (distance from the surface of a semiinfinite body); z_h, hardened-layer thickness; $\zeta = z/a$, dimensionless coordinate; $\zeta_h = z_h/a$, dimensionless hardened-layer thickness; $\tau = kt/a^2$, $\tau_r = kt_r/a^2$, $\tau_s = kt_s/a^2$, and $\Delta\tau = k\Delta t/a^2$, Fourier numbers; $\tau_{max} = \tau_s + \Delta\tau$, dimensionless time of attainment of the maximum temperature; $\delta\tau$, dimensionless step of partition of the time interval; $\Lambda = Aq_0a/K$, factor having the dimension of temperature. Subscripts: (0) and (1), rectangular and triangular forms of a pulse; s, switching off; r, rise; h, hardening; max, maximum quantities; *, dimensionless quantities.

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